Decoherence-Based Quantum Zeno Effect in a Cavity-QED System

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We present a decoherence-based interpretation for the quantum Zeno effect (QZE) where measurements are dynamically treated as dispersive couplings of the measured system to the apparatus, rather than the von Neumann's projections. It is found that the explicit dependence of the survival probability on the decoherence time quantitatively distinguishes this dynamic QZE from the usual one based on projection measurements. By revisiting the cavity-QED experiment of the QZE [J. Bernu, *et al.*, Phys. Rev. Lett, **101**, 180402 (2008)], we suggest an alternative scheme to verify our theoretical consideration that frequent measurements slow down the increase of photon number inside a microcavity due to the nondemolition couplings with the atoms in large detuning.

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Introduction – It usually follows from the von Neumann's postulate of wave packet collapse (WPC) that the frequent measurements about whether the system stays in its initial unstable state would inhibit the transitions to other states [1]. This inhibition phenomenon is now called the quantum Zeno paradox or the quantum Zeno effect (QZE). Some experiments, which claimed the verifications of the QZE for various physical systems [2–4], seemed to provide clear evidence supporting the necessariness of the WPC in the logical system of quantum mechanics. However, many physicists wondered whether the QZE phenomena were really rooted in the WPC-based measurement (or called the projection measurement) [5–12].

In the early days of the discovery of the QZE, Asher Peres demonstrated that the QZE-like phenomenon could also be explained in terms of the strong interaction between the observed system and an external agent [7]. When Itano *et al.* carried out a QZE experiment based on the theoretical proposal of Cook [8] and claimed the role of the projection measurement [2], some authors argued that no WPC really happened since the existing experimental data could also be recovered by unitary dynamic calculations without invoking the WPC [9, 10]. Furthermore, a recent experiment in cavity-QED system for freezing the growth of the photon number in a cavity was explained in terms of the WPC-based QZE [13]. It awakened us to seriously revisit the problem whether this QZE phenomenon depends on the von Neumann's postulate [14], which lies in the core of Copenhagen's quantum mechanics interpretation (QMI). We expect the similar experiment and its extension could provide an accessible way to well clarify the physical distinguishability of different QMIs in accounting for the QZE.

In this Letter, We generally describe the QZE by a unitary evolution regarding the quantum measurement as a dispersive coupling for decoherence [15, 16]. With respect to the system's eigenstates being measured, the decoherence-based quantum measurement is generally formulated by a diagonal normal operator valued in the apparatus' observable (we call it the measurement operator) [12, 17, 18]. Then we show the frequent "bang-bang" insertions of such measurement operators in the original time evolution decohere the system. These frequent measurements cancel the off-diagonal elements of the system's density matrix through the destructive interference. Therefore, the transitions among the eigenstates of the system are inhibited.

This universal proof deals with quantum measurement as a dynamic dephasing process, rather than an instantaneous collapse. Thus the measurement time is introduced as a crucial parameter to signature our decoherence-based model in contrary to the conventional WPC-based one. By re-considering the cavity-QED experiment [13] where the periodically driven cavity field is measured by the nondemolition dispersive couplings to the injected off-resonant atoms, we calculate the two-dimensional "phase diagrams" of an alternative experimental scheme with respect to the measurement time and the "bang-bang" time interval. Characterizing the dynamic nature of the QZE, the dependence of the survival probability on the measurement time explicitly reflect the experimentally testable difference between two QMIs related to the WPC and dispersive couplings respectively.

Decoherence-induced quantum Zeno effect – Now we develop a general approach for QZE based on dynamic description of quantum measurement [11, 12, 17]. The dispersive couplings of the measured system S to the apparatus A lead to a time evolution of the total system S plus A from the initial state $|\varphi(0)\rangle = \sum_j c_j |s_j\rangle \otimes |a\rangle$ to an entangled state $|\varphi(t)\rangle = M(t) |\varphi(0)\rangle \equiv \sum_j c_j |s_j\rangle \otimes |a_j\rangle$. Here $|s_j\rangle$ $(j = 1, 2, \ldots)$ serves as an orthonormal basis of the Hilbert space $\mathcal{H}_S$ of S, while $|a\rangle$ is the initial state of A. The unitary measurement operator $M(t)$ is a diagonal normal matrix with elements $M_{jj} = \exp(-i\hat{h}_j t)$ for the branch Hamiltonian $\hat{h}_j$ being a Hermitian operator on the Hilbert space $\mathcal{H}_A$ of A. The final state $|a_j\rangle \equiv |a_j(t)\rangle = \exp(-i\hat{h}_j t) |a\rangle$ of A corresponds to the system's state $|s_j\rangle$. Obviously, $M(t)$ is capable of defining a nondemolition measurement [19]. An ideal measurement could well distinguish the apparatus state

$|a_j\rangle$ from $|a_{j'}\rangle$, i.e., $\langle a_{j'}|a_j\rangle = \delta_{jj'}$. In this ideal case, the reduced density matrix of the system is depicted by $\rho_s(t) = \mathrm{Tr}_A(|\varphi(t)\rangle\langle\varphi(t)|)$ with vanishing off-diagonal elements.

$U(t)$ is defined as the unitary evolution operator of S in the absence of the above "measurement". Then we generally describe the QZE phenomenon by a unitary evolution matrix $U_c(t) = U_c(\tau, \tau_m) = [M(\tau_m)U(\tau)]^N$ (see Fig. 1) with a fixed duration $t = N\tau$. Here τ indicates a small time interval for which the system evolves freely, and a measurement with shorter time τ_m is performed at the end of each $U(\tau)$. Actually, the free evolution co-exists with the measurements through the whole QZE process, but it could be ignored when measurement is turned on since the apparatus induces a fast decoherence. An ideal measurement requires a very short τ_m, but a finite τ_m will reflect the dynamic feature of the realistic measurements. Usually, $U(\tau)$ does not commutate with $M(\tau_m)$ so that it can induce the transitions among states $|s_j\rangle$. We re-write $U_c(t)$ as a N-multi-product

$$U_c(\tau, \tau_m) = \left[\prod_{n=1}^{N} U_n(\tau)\right] M^N, \tag{1}$$

where the factors $U_n(\tau) = M^n U(\tau) M^{-n}$ for $M \equiv M(\tau_m)$ and $n = 1, 2, ..., N$. For a very short τ or a very large N, it could be approximated as $U_n(\tau) \simeq 1 - i\tau M^n H M^{-n} \equiv 1 - i\tau H_n$. If M is not degenerate, we have

$$U_c(\tau, \tau_m) \simeq \left(1 - itH_d - i\frac{t}{N}S\right) M^N, \tag{2}$$

where A_d and A_{off} denote the diagonal and off-diagonal parts of matrix A, respectively. The summation $S = \sum_n (H_n)_{\mathrm{off}}$ is convergent as $N \to \infty$ or $\tau \to 0$ for fixed $t = N\tau$, since $S = \sum_{j\neq j'} \Lambda_{jj'} H_{jj'} |s_j\rangle\langle s_{j'}|$, where

$$\Lambda_{jj'} = \frac{\sin(\frac{1}{2}\tau_m N \Delta_{jj'})}{\sin(\frac{1}{2}\tau_m \Delta_{jj'})} e^{-i\tau_m(N+1)\Delta_{jj'}/2} \tag{3}$$

for $\Delta_{jj'} = \hat{h}_j - \hat{h}_{j'}$. $\Lambda_{jj'}$ is a finite number when $\Delta_{jj'} \neq 0$, then in the large-N limit, the QZE is achieved as

$$\lim_{N\to\infty} U_c(\tau, \tau_m) \to e^{-iH_d t}[1 - i\mathcal{O}(\frac{t}{N})]M^N. \tag{4}$$

Therefore, the time evolution with very frequent M-kicks will keep the system in its initial state because $U_c(\tau, \tau_m)$ approaches a diagonalized unitary matrix $\exp(-iH_d t)$.

This argument generally proves the QZE in a dynamic version. Thus the frequent measurements (for $N \to \infty$) based on the decoherence model indeed result in the QZE even though no WPC is used. We remark that the similar arguments for the QZE have been given by making use of the von Neumann's quantum ergodic theorem [20].

Cavity-QED setup for testing decoherence-based quantum Zeno effect – The experiment based on high-Q superconducting cavity has explicitly demonstrated the increase of the photon number inside the cavity is suppressed by the continuous measurements [13]. In this experiment, a series of microwave pulses resonant with the cavity are injected into the cavity, which corresponds to the U-process; between every two adjacent pulses an ensemble of off-resonant atoms are sent into the cavity to probe the average photon number, playing the part of the M-process. A single QND probe is actually a dynamic process and changes the cavity field by a phase factor instead of its photon number. Even we do not read out the photon number after each probe, the QND coupling of the cavity field to the off-resonant atom can result in the phase random in the accumulation of these phase factors thus leads to freezing the photon number in its initial state. We propose an alternative cavity-QED scheme to verify this illustration.

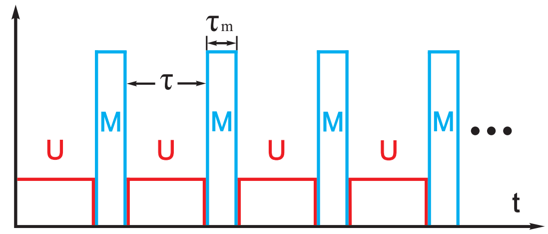

FIG. 1: Controlled evolution process containing N unitary evolution U -processes and N dynamic measurement M-processes. The y-axis represents the strength of the interaction.

Let the cavity be initially prepared in the vacuum state $|0\rangle$ with an ensemble of off-resonant atoms located in it. Then classical driving laser pulses are sequentially injected into the cavity. Each pulse is applied for a duration τ. This unitary evolution of the cavity field is described by the Hamiltonian

$$H_U(t) = \omega a^\dagger a + f e^{-i\omega_F t} a^\dagger + h.c., \tag{5}$$

where ω is the frequency of the cavity, f and ω_F the strength and the frequency of the driving field respectively, a and $a^\dagger$ the annihilation and creation operators of the cavity field. The driving pulse is peaked at the frequency resonant with the cavity, i.e., $\omega_F \approx \omega$. Compared to the strength of the driving field, the interaction between the atom and the cavity field is rather weak, and thus can be omitted when the pulse is switched on. In the interval when we turn off the driving field, the atom-field interaction becomes important. Since the energy level spacing ω_a of the atom and the frequency ω of the cavity are largely detuned, adiabatic elimination results in an effective measurement Hamiltonian

$$H_M = \frac{g^2}{\Delta} a^\dagger a \left(|+\rangle\langle+| - |-\rangle\langle-|\right). \tag{6}$$

Here $|\pm\rangle$ are the two atomic energy levels, g the vacuum Rabi frequency defining the atom-cavity coupling and $\Delta = \omega - \omega_a$ the atom-cavity detuning. The unitary evolution dominated by H_M is regarded as a QND

measurement, for the atom records the information of the photon number of the cavity field by its phase of the $|\pm\rangle$ superposition. The whole experimental procedure consists of a series of dynamic processes described by H_U and H_M alternatively the same as demonstrated in Fig. 1, but the strengths of U and M processes are reversed. The probe of the photon number is only carried out after the last driving pulse.

Free evolution and decoherence-based measurement – The time evolution of the cavity field governed by $H_U(\tau)$ is described by phase-modulated displacement operator

$$U(\tau) = e^{i\omega a^\dagger a\tau} e^{i\phi(\tau)} D[\alpha(\tau)], \tag{7}$$

where $D[\alpha(\tau)] = \exp[\alpha(\tau) a^\dagger - \alpha^*(\tau) a]$ with the displacement parameter $\alpha(\tau) = [\exp(-i\delta\tau) - 1] f/\delta$, and the phase factor is $\phi(\tau) = (\sin\delta\tau - \delta\tau) f^2/\delta^2$, $\delta = \omega_F - \omega$. Here the Wei-Norman algebra method [21] is used in deriving $U(\tau)$.

In a cavity in the vacuum state $|0\rangle$, the atom is initially prepared in the superposition state $|\phi(0)\rangle = (|+\rangle + |-\rangle)/\sqrt{2}$. After the first driving pulse applied for time τ, the total system evolves into $|\psi(\tau)\rangle = |\phi(0)\rangle \otimes |\alpha(\tau)\exp(-i\omega\tau)\rangle$. We can see that the average photon number $\bar{n} = |\alpha(\tau)|^2 \approx f^2\tau^2$ quadratically depends on τ, for τ is a sufficiently short interval. Then the pulse is turned off and the atom-cavity field interaction H_M dominates the unitary evolution by $M(\tau_m) = \exp(-i\tau_m H_M)$ for the measurement interval τ_m. After the first measurement, the state $|\psi(\tau)\rangle$ evolves into an atom-cavity field entangled state,

$$|\psi(\tau+\tau_m)\rangle = \frac{1}{\sqrt{2}} e^{i\phi(\tau)} \sum_{j=\pm} |j\rangle \otimes |\alpha_j\rangle, \tag{8}$$

with $\alpha_\pm \equiv \alpha(\tau)\exp\left(-i\omega\tau \mp ig^2\tau_m/\Delta\right)$. The average photon number does not change due to the QND nature of the measurement, but the cavity field acquires different phases corresponding to the two atomic states.

Continuous measurement process for QZE – During the free evolution, we insert the decoherence-based measurements for N times at instants $n\tau$ $(n = 1, 2, \ldots, N)$. Mathematically, we apply $[M(\tau_m) U(\tau)]^N$ to the initial state, and then the quantum state evolves into $|\psi_N\rangle \equiv |\psi[N(\tau+\tau_m)]\rangle = \sum_{j=\pm} [M_j(\tau_m) U(\tau)]^N |j\rangle \otimes |0\rangle$. Here $M(\tau_m)$ acts on the cavity field as two operators $M_\pm(\tau_m) = \exp\left(\mp i\xi_m a^\dagger a\right)$ corresponding to the two atomic states respectively, where $\xi_m = g^2\tau_m/\Delta$. From the calculations of the explicit expression for $[M_\pm(\tau_m) U(\tau)]^N$, we finally obtain the evolution wavefunction

$$|\psi_N\rangle = \sum_{j=\pm} \frac{e^{i\phi_j}}{\sqrt{2}} |j\rangle \otimes \left|\alpha_{jN} e^{-i\omega t}\right\rangle, \tag{9}$$

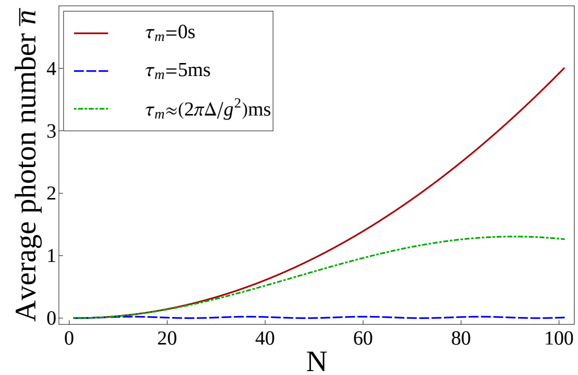

FIG. 2: (color online) Average photon number $\bar{n}$ as a function of the pulse number N. We choose $g^2/\Delta = 10$kHz, $\delta = 0.5$Hz, $f = 400$Hz, and $\tau = 50\mu$s. Without the QND probe, $\bar{n}$ grows quadratically with N (red solid line). The QZE emerges as $\bar{n}$ is frozen at zero with $\tau_m = 5$ms (bule dashed line). If the measurement time is chosen specifically at $\tau_m = (2\pi\Delta/g^2 + 3.5)\mu$s, $\bar{n}$ increases obviously (green dashdotted line) which is not explained in terms of the WPC interpretation.

where $\phi_\pm = N\phi(\tau) + \theta_\pm(N)$, and

$$\begin{aligned}\theta_\pm(N) &= \pm\frac{|\alpha(\tau)|^2}{2}\frac{N\sin\xi_m - \sin(N\xi_m)}{1-\cos\xi_m},\\ \alpha_{\pm N} &= \alpha(\tau) e^{\mp i(N+1)\xi_m}\frac{\sin(N\xi_m/2)}{\sin(\xi_m/2)}.\end{aligned}$$

Accordingly the average photon number is calculated as

$$\bar{n} = |\alpha(\tau)|^2 \frac{\sin^2(N\xi_m/2)}{\sin^2(\xi_m/2)}. \tag{10}$$

We can see in the continuous measurement limit, i.e., $\tau \to 0$, $|\alpha(\tau)|^2 \approx f^2\tau^2$. Except for certain measurement time interval τ_m^* chosen as $\xi_m = g^2\tau_m^*/\Delta = 2k\pi$, with k integral, $\bar{n}$ approaches zero with τ decreasing.

As illustrated in Fig.2, $\bar{n}$ shows the similar inhibition phenomenon (blue dashed line) to Ref. [13], with τ chosen as 50μs. The reason for the photon number ceasing increase is that the dynamic measurements interrupt the coherent accumulation of photons by adding a phase factor to the cavity field corresponding to ξ_m. The total phase factor after N-times measurement destroys the quantum interference of the cavity field, thus leads to the QZE. This deocoherence-based process in the existing experiment [13] reveals that the QZE can be completely interpreted from the dynamic aspect. To compare with the situation with only free evolution and no measurements, we set the atom-cavity coupling $g = 0$, and $\bar{n}$ is also depicted in Fig. 2 (red solid line), which indeed grows quadratically with $t = N\tau$.

The above argument is coincident with the existing experimental data, but this theoretical description implies the difference between the dynamical measurement and the projection one. We notice that, when the measurement time interval is set at critical values $\tau_m^* = 2k\pi\Delta/g^2$

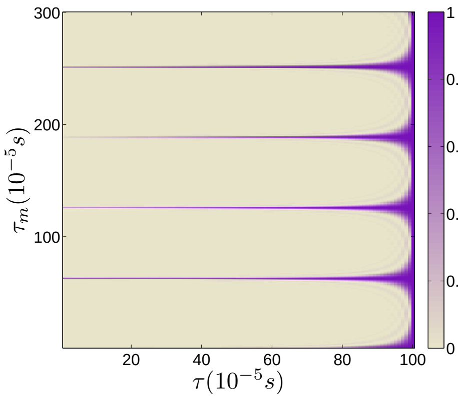

FIG. 3: The average photon number as a function of the free evolution time interval τ and the measurement time interval τ_m, where $g^2/\Delta = 10$kHz, $\delta = 0.5$Hz, $f = 400$Hz, and the total free evolution time t is fixed at 1ms. The result is normalized by the maximum.

$(k = 1, 2, 3, ...)$, $\bar{n}$ is no longer bounded and increases linearly with N. In Fig. 2, $\bar{n}$ increases clearly shown as the green dashdotted line, with τ_m chosen around τ_m^* as $\left(2\pi\Delta/g^2 + 3.5\right)\mu$s. Fixing the total free evolution time t, we illustrate the variation of the average photon number in the cavity field corresponding to the time interval τ and τ_m in Fig. 3. For a given τ_m far from the critical value τ_m^*, $\bar{n}$ approaches to zero as τ decreases, which recovers the conventional QZE phenomenon based on the projection measurement. However, $\bar{n}$ mounts up evidently when τ_m approaches to τ_m^*. This τ_m-dependent decoherernce-based QZE could not be predicted by the WPC interpretation, but can be testified by the realizable cavity-QED experiment. If we observe the rise up of the average photon number at certain τ_m^* in continuous measurement limit, then we can conclude that the dynamic measurement model is more compatible with the physical reality in comparison with the projection measurement in respect of the QZE.

Conclusion – In this Letter, we provided a general algebraic proof that QZE could be induced by frequent decoherence-based measurements, which are unitary processes without reference to the WPC postulate (projection measurement). This approach essentially shows the general QZE phenomenon can be explained independent of the quantum mechanics-interpretation for the measurement. Projection measurement provides us a neat description of the QZE, beyond which, the decoherenced-based model contains more physical detail. In tht quantum open system, the same model can be extended to predict the QZE or anti-QZE [22]. Associated with a recent cavity QED experiment [13], we predict an observable effect of the decoherence-based measurements to distinguish it from the one based on projection measurement: the survival probability after finite N measurements will explicitly depend on the measurement time even in the continuous limit. At certain critical measurement times, the survival probability will deviate from its initial value predicted in the WPC-based explanation of the QZE.

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